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Steady-State Configuration of an Underwater Suspended Bipod Cable System

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Underwater suspended bipod cable systems are utilized to moor oceanographic instrumentation. An analysis to determine the steady-state configuration of an underwater-suspended bipod cable system subject to buoyant lift forces as well as current drag forces is presented. It is assumed that the cables are neutrally buoyant or can be made neutrally buoyant by the addition of buoyancy and that the effects of tangential drag on each cable are negligible. The equations used to find the equilibrium position of the buoyed joint and also the tensions in each mooring cable are of two kinds: three force equilibrium equations at the buoyed joint and two compatibility equations for the displaced bipod system. These equations are sufficient to solve for the five unknowns, namely, the displacement of the buoyed end in three directions and the two cable tensions. A computer program was developed to solve the system of equations. Iterations are performed until a convergence is obtained on an equilibrium position and cable tensions. As an example, a bipod system 10,000 ft high with a 20,000 lb buoyant force under installed conditions was considered. The flow direction was taken at an angle of 30 deg with the normal of the plane containing the system. A linear velocity profile ranging from 1 ft/s at the top to 0 at the bottom was found to cause a 670 ft horizontal and 153 ft vertical deflection of the buoyed joint.

Structure

THE cable system analyzed here comprises two flexible cables with their lower ends separated and anchored on the ocean bottom, and their upper ends joined together and attached to a subsurface buoy, forming an inverted "V," as shown in Fig. 1.

Geometric Considerations

The cable system will be referred to a Cartesian coordinate system. Vectorial representation describes the cable configuration. Each cable has an average unit tangent vector $\bar{t}_i$ (Fig. 1), which is the vector given by the relative position of the cable endpoints. The subscripts indicate the cable number.

Writing the vector equation giving x, y, and z components, respectively, of the undeflected cable system we have, for cable 1

$$\bar{t}_1 = 0, \frac{d_1}{\sqrt{d_1^2+h_1^2}}, \quad \frac{h_1}{\sqrt{d_1^2+h_1^2}} \tag{1}$$

and similarly for cable 2

$$\bar{t}_2 = 0, \quad \frac{-d_2}{\sqrt{d_2^2+h_2^2}}, \quad \frac{h_2}{\sqrt{d_2^2+h_2^2}} \tag{2}$$

where

h_i = height of cable i
d_i = distance to center of cable system from anchor point of cable i

In Eqs. (1) and (2), the quantities d_i and h_i will not carry the subscripts in remaining calculations. Instead, an analysis will be performed on a symmetrical bipod, keeping in mind that the equations are adequate for an unsymmetrical system as well as if subscripts are retained.

We will now introduce a deflection vector $\bar{\Delta}$, (Fig. 1), resulting from a current having a velocity vector $\bar{V}$. This vector is given by

$$\bar{\Delta} = \Delta X, \Delta Y, -\Delta Z \tag{3}$$

where ΔX, ΔY, and ΔZ are the unknown deflection components which will be solved for in the following analysis.

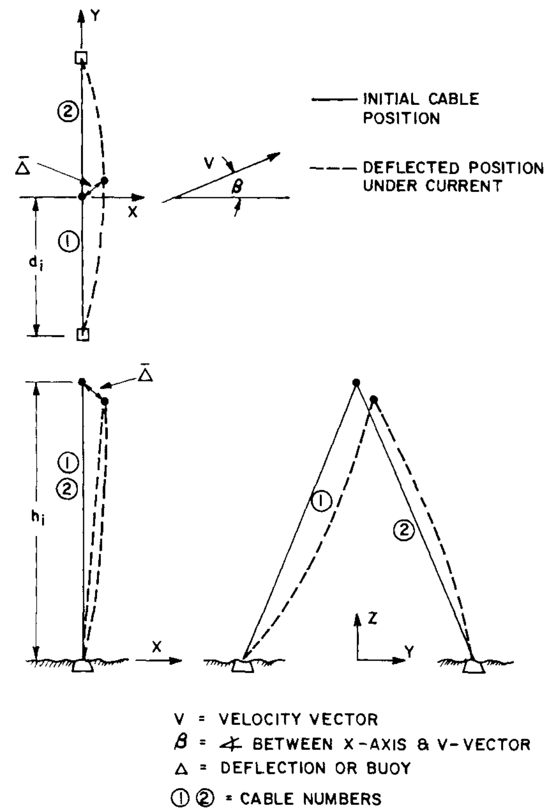

Fig. 1 Cable configuration.

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Figure 1 shows the directions for positive deflections. The sign convention shown in this figure indicates the need for a negative ΔZ component.

Let the velocity vector $\bar{V}$ be given as

$$\bar{V} = V(s)\cos\beta, V(s)\sin\beta, 0 \tag{4}$$

where

β = angle the velocity vector makes with the x axis, measured counterclockwise from the positive direction of the axis (Fig. 1)

$V(s)$ = the magnitude of velocity given as a function of s, the distance along the cable

Having given the buoyed joint a deflection $\bar{\Delta}$, the average unit tangent vectors must be redefined. That is, Eqs. (1) and (2) become

$$\bar{t}_1 = \frac{\Delta X}{\sqrt{h^2+d^2}}, \quad \frac{\Delta Y + d}{\sqrt{h^2+d^2}}, \quad \frac{h - \Delta Z}{\sqrt{h^2+d^2}} \tag{5}$$

and

$$\bar{t}_2 = \frac{\Delta X}{\sqrt{h^2+d^2}}, \quad \frac{\Delta Y - d}{\sqrt{h^2+d^2}}, \quad \frac{h - \Delta Z}{\sqrt{h^2+d^2}} \tag{6}$$

The quantity $\sqrt{d^2+h^2}$ is used to unitize the $\bar{t}_i$ vectors instead of the exact value $\sqrt{\Delta X^2 + (\Delta Y + d)^2 + (h - \Delta Z)^2}$. It is assumed the deflections are small compared to the length of cable and the difference between the two quantities is negligible.

In calculating the drag forces on a cable, the tangential and normal velocity components are needed. The tangential component is found by taking the scalar product of the velocity vector [Eq. (4)] with the tangent vector for each cable.

Therefore V_t^i = magnitude of the tangential velocity of the ith cable will be given by

$$V_t^i = \bar{V} \cdot \bar{t}_i \tag{7}$$

Using Eq. (7), the magnitudes of the tangential velocity components for each cable are

$$V_t^1 = \frac{V}{\sqrt{h^2+d^2}} [\Delta X \cos\beta + (\Delta Y + d)\sin\beta] \tag{8}$$

and

$$V_t^2 = \frac{V}{\sqrt{h^2+d^2}} [\Delta X \cos\beta + (\Delta Y - d)\sin\beta] \tag{9}$$

The tangential vector of cable i is

$$\bar{V}_t^i = V_t^i (\bar{t}_i) \tag{10}$$

For each cable this becomes

$$\bar{V}_t^1 = \frac{V}{h^2+d^2} [\Delta X \cos\beta + (\Delta Y + d)\sin\beta] \times (\Delta X \hat{i} + (\Delta Y + d)\hat{j} + (h - \Delta Z)\hat{k}] \tag{11}$$

and

$$\bar{V}_t^2 = \frac{V}{h^2+d^2} [\Delta X \cos\beta + (\Delta Y - d)\sin\beta] \times (\Delta X \hat{i} + (\Delta Y - d)\hat{j} + (h - \Delta Z)\hat{k}] \tag{12}$$

where $\hat{i}$, $\hat{j}$, and $\hat{k}$ are unit vectors along x, y, and z axis, respectively.

To find the normal component, we use simple vector addition

$$\bar{V} = \bar{V}_n^i + \bar{V}_t^i \quad \text{or} \quad \bar{V}_n^i = \bar{V} - \bar{V}_t^i \tag{13}$$

Using Eq. (13) the normal velocity of each cable becomes

$$\bar{V}_n^1 = V\left\{\left[\cos\beta - \frac{(\Delta X\cos\beta + (\Delta Y + d)\sin\beta)\Delta X}{h^2+d^2}\right]\hat{i} + \left[\sin\beta - \frac{(\Delta X\cos\beta + (\Delta Y + d)\sin\beta)(\Delta Y + d)}{h^2+d^2}\right]\hat{j} - \left[\frac{(\Delta X\cos\beta + (\Delta Y + d)\sin\beta)(h - \Delta Z)}{h^2+d^2}\right]\hat{k}\right\} \tag{14}$$

and for cable 2

$$\bar{V}_n^2 = V\left\{\left[\cos\beta - \frac{(\Delta X\cos\beta + (\Delta Y - d)\sin\beta)\Delta X}{h^2+d^2}\right]\hat{i} + \left[\sin\beta - \frac{(\Delta X\cos\beta + (\Delta Y - d)\sin\beta)(\Delta Y - d)}{h^2+d^2}\right]\hat{j} - \left[\frac{(\Delta X\cos\beta + (\Delta Y - d)\sin\beta)(h - \Delta Z)}{h^2+d^2}\right]\hat{k}\right\} \tag{15}$$

Absolute Unit Vector

For the solution of the bipod system, three equilibrium equations at the buoyed joint will be used. These equations require the unit tangent vector of each cable at that point. The average tangent vectors already defined by Eqs. (5) and (6) do not take into account the bowing action of the cable under current load. This current load will change the direction of tangency from that given by the average tangent vector. Therefore, another tangent vector must be defined.

Let $\bar{\tau}_i$, illustrated in Fig. 2, be the unit tangent vector of the ith cable at any point along the cable. In Fig. 2 is also shown ϕ, the angle between the average tangent vector $\bar{t}_i$ and the absolute tangent vector $\bar{\tau}_i$. It will be assumed the cable bowing occurs in a plane defined by $\bar{t}_i$ and the normal velocity component to a cable. The calculation of the angle ϕ now becomes a two-dimensional problem.

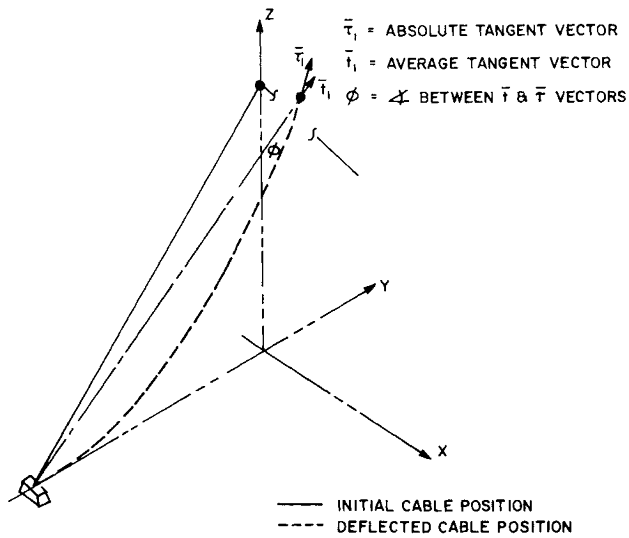

Fig. 2 Tangent vectors.

Let $\bar{\tau}_i$, illustrated in Fig. 2, be the unit tangent vector of the i_{th} cable at any point along the cable. In Fig. 2 is also shown ϕ, the angle between the average tangent vector $\bar{t}_i$ and the absolute tangent vector $\bar{t}_i$. It will be assumed the cable bowing occurs in a plane defined by $\bar{t}_i$ and the normal velocity component to a cable. The calculation of the angle ϕ now becomes a two-dimensional problem.

General Solution of a Two-Dimensional Cable Model[1]

Referring to a two-dimensional cable model, as shown in Fig. 3a, we may investigate the bowing effect. An element of cable Δs is shown in Fig. 3b. The forces acting on this cable, element after installation and load are: the weight of the cable, the tensions at each end, and the normal and tangential drag forces. It has been assumed the cable will be neutrally buoyant and the effects of tangential drag will be negligible. Eliminating these, the system of forces in Fig. 3b reduce to those shown in Fig. 4.

Using Fig. 4, we sum forces in the tangential direction

$$T+\Delta T-T=0 \tag{16}$$

Dividing through by Δs and taking the limit as Δs approaches zero we obtain:

$$\lim_{\Delta s\to 0}\frac{(T+\Delta T-T)}{\Delta s}=\lim_{\Delta s\to 0}\frac{\Delta T}{\Delta s}=\frac{dT}{ds}=0 \tag{17}$$

Equation (17) implies that the change in tension along our weightless cable model, with tangential drag neglected, is zero or the tension is constant along the cable.

Summing forces in the normal direction, neglecting second-order terms, gives a differential equation of the form

$$-D_n+T\frac{d\gamma}{dS}=0 \tag{18}$$

In general, the normal drag force per unit length of cable may be given by

$$D_n=½\rho c(V_n^i)^2 d \tag{19}$$

where ρ is the density of liquid, c is the coefficient of drag, V_n^i is defined as before, and d equals the diameter of cable used.[2] For instances where buoyancy is added, a composite coefficient of friction and diameter which accounts for the affects of buoyancy must be applied. Equation (18) becomes

$$\frac{d\gamma}{ds}=\frac{½\rho c(V_n^i)^2 d}{T} \tag{20}$$

This is an expression for the change in the tangent angle along the cable length ds.

Simplifying Eq. (20) by letting

$$K=\frac{½\rho cd}{T} \tag{21}$$

we have

$$d\gamma=K(V_n^i)^2 dS \tag{22}$$

Integrating Eq. (22) along the cable length L gives

$$\int_{\phi_1}^{\phi_2} d\gamma=\phi_2-\phi_1=K\int_0^L (V_n^i)^2 ds \tag{23}$$

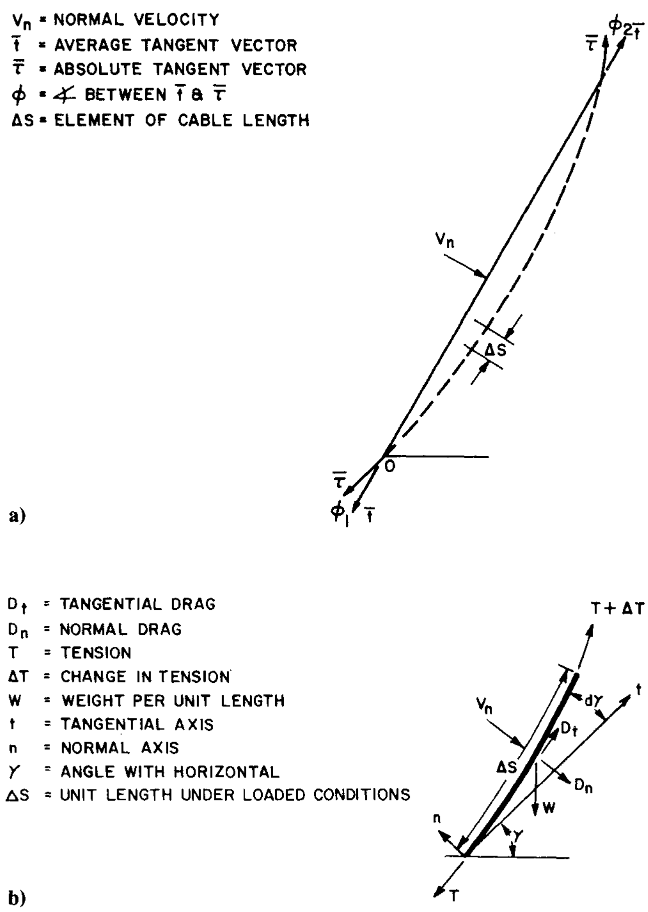

Fig. 3 Two-dimensional cable.

In Eq. (23), V_n^i is the normal velocity variation given as a function of s, the distance along the cable. To obtain an approximate expression for ϕ, one constant must be determined.† This is found by taking moments about a line through the lower support and normal to the plane defined by the average $\bar{t}_i$ and velocity component $\bar{V}_n^i$ normal to $\bar{t}_i$ as shown in Fig. 3a.

$$M_0=-k\int_0^L (V_n(s))^2 s ds+TL\sin\phi_2=0 \tag{24}$$

where from Eq. (21), $k=KT$. Setting Eq. (24) equal to zero gives

$$\sin\phi_2=\frac{K}{L}\int_0^L (V_n(s))^2 s ds \tag{25}$$

or

$$\phi_2=\sin^{-1}\left(\frac{K}{L}\int_0^L (V_n(s))^2 s ds\right) \tag{26}$$

Rewriting Eq. (23) and replacing ϕ_1 with $\phi(s)$ and indicating ϕ as a function of s we have

$$\phi(s)=\sin^{-1}\left\{\frac{K}{L}\int_0^L [V_n(s)]^2 s ds\right\}-K\int_s^L [V_n(s)]^2 s ds \tag{27}$$

†A more exact expression would be obtained utilizing $V_n^i(s)=V_n^i\cos\phi(s)-V_t^i\sin\phi(s)$ where $\phi(s)$ is the angle between $\bar{\tau}(s)$ and $\bar{t}$.

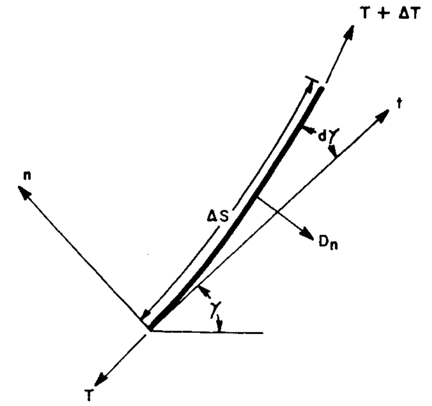

Fig. 4 Cable element.

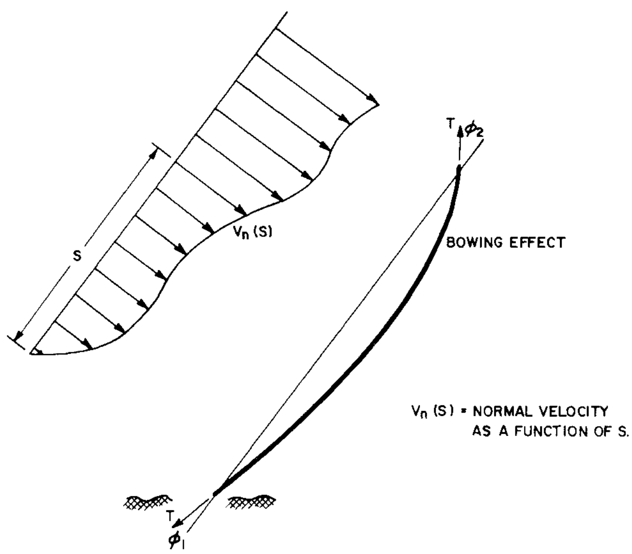

Fig. 5 Cable equilibrium.

Having calculated the ϕ angle it is now possible to write an expression for $\bar\tau$ (see Fig. 5). From vector addition,

$$\bar\tau_i = \bar t_i \cos\phi - (\bar V_n^i / V_n^i)\sin\phi \tag{28}$$

$\bar V_n^i$ is given by Eqs. (14) and (15). For the first approximation of its magnitude, assuming ΔX, $\Delta Y = 0$ gives

$$V_n^i = V\sqrt{1 - \frac{d^2\sin^2\beta}{d^2+h^2}} \tag{29}$$

Substituting in Eq. (28) the quantities t_i, V_n^i, and $\bar V_n^i$ the two tangential vectors are obtained:

$$\bar\tau_1 = \left\{ \frac{\cos\phi\Delta X}{\sqrt{h^2+d^2}} - \frac{V}{V_n^1}\sin\phi\left[\cos\beta - \frac{(\Delta X\cos\beta + (\Delta Y + d)\sin\beta)X}{h^2+d^2}\right]\right\}\hat i$$

$$+ \left\{ \frac{\cos\phi(\Delta Y + d)}{\sqrt{h^2+d^2}} - \frac{V}{V_n^1}\sin\phi \times \left[\sin\beta - \frac{(\Delta X\cos\beta + (\Delta Y + d)\sin\beta)(\Delta Y + d)}{h^2+d^2}\right]\right\}\hat j$$

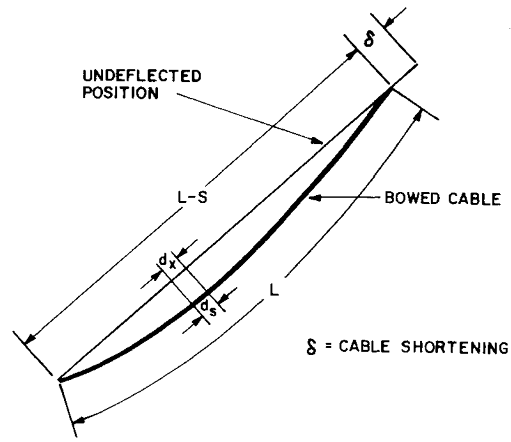

Fig. 6 Cable shortening.

$$+ \left\{ \frac{\cos\phi(h-\Delta Z)}{\sqrt{h^2+d^2}} - \frac{V}{V_n^1}\sin\phi \times \frac{(\Delta X\cos\beta + (\Delta Y + d)\sin\beta)(h-\Delta Z)}{h^2+d^2}\right\}\hat k \tag{30}$$

To obtain an expression for $\bar\tau_2$, replace the quantity $(\Delta Y + d)$ in the above expression by $\Delta Y - d$.

Joint Compatibility

To establish the position of the buoyed joint, consideration must be given to the effects of cable bowing. As loading occurs, the distance between cable endpoints decreases. The amount of decrease will be defined as δ and is shown in Fig. 6. An element of length dx along the cable, shown in the figures is given by

$$dx = \cos\phi ds \tag{31}$$

where ϕ is the angle between $\bar t$ and the actual tangent vector at x.

Expansion of cosine and substitution in Eq. (31) gives

$$dx = \left(1 - \frac{\phi^2}{2}\right)ds \tag{32}$$

Integrating both sides from zero to $L-\delta$ on the left and zero to L, the length of cable, on the right we obtain:

$$\int_0^{L-\delta} dx = \int_0^L \left(1 - \frac{\phi^2}{2}\right)ds$$

or

$$\delta = \int_0^L \frac{\phi^2}{2}ds \tag{33}$$

Equation (27) gives ϕ as a function of s. Substituting this into Eq. (33)

$$\delta = \tfrac{1}{2}\int_0^L \left\{\sin^{-1}\frac{K}{L}\int_0^L [V_n(s)]^2 s ds - K\int_s^L [V_n(s)]^2 ds\right\}^2 ds \tag{34}$$

which is the amount of shortening in cable endpoints due to velocity distribution $V_n(s)$. As an example to demonstrate the use of Eq. (34) we consider a linear normal velocity distribution. Let the variation in velocity be one which ranges from some magnitude at the top of the array to zero at the bottom. Velocity may then be written as

$$V_n(s) = V_n(\mathrm{VPS})s \qquad (35)$$

where V_n is the magnitude of normal velocity at the top of the array, VPS is the velocity profile slope, and s the distance along the cable. VPS would be 1/10,000 if the velocity ranged from 1 ft/s to 0 in a 10,000 ft deep bipod system. Substituting $V_n(s)$ into Eq. (34) yields

$$\delta = \tfrac{1}{2}\int_0^L \left\{\theta_f - \alpha\int_0^L s^2 \mathrm{d}s\right\}^2 \mathrm{d}s \qquad (36)$$

where

$$\theta_f = \sin^{-1}\frac{\alpha}{L}\int_0^L s^3 \mathrm{d}s = \sin^{-1}\frac{\alpha L^3}{4}$$

and

$$\alpha = K\,|V_n|^2 \mathrm{VPS}^2 \qquad (37)$$

Simplifying Eq. (36)

$$\delta = \tfrac{1}{2}\int_0^L \left(\theta_f - \frac{\alpha s^3}{3}\right)^2 \mathrm{d}s \qquad (38)$$

or finally upon integrating

$$\delta = \frac{L}{2}\left(\theta_f^2 - \frac{\alpha L^3}{6} + \frac{\alpha^2 L^6}{63}\right) \qquad (39)$$

This is the shortening of the distance between endpoints of a cable under a linear distribution of velocity. The value of this shortening will exceed the actual value by an amount equal to the elastic stretch in the cable. Its magnitude being $\Delta TL/EA$, where ΔT= the change in cable tension from the original undisturbed configuration, L= length, E= modulus of elasticity, and A = cross-sectional area of the cable.[3]

Having calculated the amount of shortening of each cable, it is possible to write two vector equations for the compatibility of the buoyed joint:

$$\bar{t}_1(L-\delta_1) = \Delta X\hat{i} + (d+\Delta Y)\hat{j} + (h-\Delta Z)\hat{k} \qquad (40)$$

and

$$\bar{t}_2(L-\delta_2) = \Delta X\hat{i} + (\Delta Y-d)\hat{j} + (h-\Delta Z)\hat{k} \qquad (41)$$

Taking the square of both sides gives for cable 1

$$(L-\delta_1)^2 = \Delta X^2 + (d+\Delta Y)^2 + (h-\Delta Z)^2 \qquad (42)$$

and for cable 2

$$(L-\delta_2)^2 = \Delta X^2 + (\Delta Y-d)^2 + (h-\Delta Z)^2 \qquad (43)$$

Equilibrium and Compatibility Equations

In the bipod cable system there are five unknowns which must be solved, that is, three unknown deflections in the x, y, and z directions and two unknown tensions in cables 1 and 2. The equations to be used are three equilibrium equations at the buoyant joint and two compatibility equations of the displacement.

For the equilibrium of the buoy, the forces due to buoy drag and buoyancy must be defined. The vector describing these forces is given by:

$$\bar{F} = F_D\cos\beta\hat{i} + F_D\sin\beta\hat{j} + F_B\hat{k} \qquad (44)$$

where
F_D = drag force
F_B = buoyant force
β = angle made with positive x axis by velocity vector (see Fig. 1)

Now it is possible to write the vector equilibrium equation for the buoyed joint. That is,

$$\bar{F} - T_1\bar{\tau}_1 - T_2\bar{\tau}_2 = 0 \qquad (45)$$

where $\bar{\tau}_1$ and $\bar{\tau}_2$ are the unit tangent vectors at the joint and T_1 and T_2 are the magnitudes of the tensions in cables 1 and 2. This vector equation represents three equations, i.e., equilibrium in the x, y, and z direction as defined in Fig. 1.

The two remaining equations will come from the compatibility given by Eqs. (42) and (43). Upon expansion of Eq. (42) the following is obtained

$$L^2 - 2\delta_1 L + \delta_1^2 = \Delta X^2 + d^2 + 2d\Delta Y + \Delta Y^2 + h^2 - 2h\Delta Z + \Delta Z^2 \qquad (46)$$

but,

$$L^2 = h^2 + d^2$$

Thus, Eq. (46) becomes

$$-2\delta_1 L + \delta_1^2 = \Delta X^2 + 2d\Delta Y + \Delta Y^2 - 2h\Delta Z + \Delta Z^2 \qquad (47)$$

The higher order terms of deflection (i.e., ΔX^2, ΔY^2, and ΔZ^2) will be neglected in Eq. (41). This gives

$$\delta_1 L = d\Delta Y - h\Delta Z \qquad (48)$$

Similarly for Eq. (43) which gives

$$\delta_2 L = d\Delta Y + h\Delta Z \qquad (49)$$

Rewriting Eq. (45) in full we obtain

$$F_D\cos\beta - \frac{T_1}{\tau_1}\left\{\frac{\cos\phi\Delta X}{\sqrt{h^2+d^2}} - \frac{V}{V_{n_1}}\sin\phi\left[\cos\beta - \frac{(\Delta X\cos\beta + (\Delta Y+d)\sin\beta)\Delta X}{h^2+d^2}\right]\right\} - \frac{T_2}{\tau_2}\left\{\frac{\cos\phi\Delta X}{\sqrt{h^2+d^2}} - \frac{V}{V_{n_2}}\sin\phi\left[\cos\beta - \frac{(\Delta X\cos\beta + (\Delta Y+d)\sin\beta)\Delta X}{h^2+d^2}\right]\right\} = 0 \qquad (50)$$

$$F_D\sin\beta - \frac{T_1}{\tau_1}\left\{\frac{\cos\phi\Delta X}{\sqrt{h^2+d^2}} - \frac{V}{V_{n_1}}\sin\phi\left[\cos\beta - \frac{(\Delta X\cos\beta + (\Delta Y+d)\sin\beta)(\Delta Y+d)}{h^2+d^2}\right]\right\} - \frac{T_2}{\tau_2}\left\{\frac{\cos\phi\Delta X}{\sqrt{h^2+d^2}} - \frac{V}{V_{n_2}}\sin\phi\left[\cos\beta - \frac{(\Delta X\cos\beta + (\Delta Y+d)\sin\beta)(\Delta Y+d)}{h^2+d^2}\right]\right\} = 0 \qquad (51)$$

$$F_B - \frac{T_1}{\tau_1}\left[\frac{\cos\phi(h-\Delta z)}{\sqrt{h^2+d^2}} - \frac{V}{V_{n_1}}\sin\phi \right.$$

$$\left. \times \frac{(\Delta X\cos\beta + (\Delta Y + d)\sin\beta)(h-\Delta Z)}{h^2+d^2}\right]$$

$$-\frac{T_2}{\tau_2}\left[\frac{\cos\phi(h-\Delta Z)}{\sqrt{h^2+d^2}} - \frac{V}{V_{n_2}}\sin\phi \right.$$

$$\left. \times \frac{(\Delta X\cos\beta + (\Delta Y - d)\sin\beta)(h-\Delta Z)}{h^2+d^2}\right] = 0 \qquad (52)$$

These last five equations (48-52) are to be solved to obtain the deflection of the buoyed joint and the unknown cable tensions.

The method described above provides equations governing the equilibrium of the cable system endpoints in terms of the distributed loading on the moorings. Skop and O'Hara[4] have treated a similar problem; their "Method of Imaginary Reactions" requires calculation of internal element conditions.

Computer Solution

From the complexity of Eqs. (48-52), a general explicit solution is not possible. Therefore, a computer program has been developed which will solve the equations using iteration. As in all iteration processes, a first approximation of the unknown quantities is needed. It is left to the analyzer to give these approximations as data in the program.

For illustration, one buoy configuration, having a buoyed joint 10,000 ft high and legs at a 45 deg angle to the horizontal, was found to deflect 670 ft in the X direction, 23 ft in the Y direction, and 153 ft in the Z direction. This was due to a velocity profile slope (VPS) of 1/10,000, an angle β of 30 deg, and a buoyant force at the apex joint of 20,000 lb.

Summary

Fundamental equations describing a cable bipod system have been presented to describe steady-state equilibrium position under current loads. Effects due to cable bowing, cable extensibility, and varying velocity profiles or current directions were incorporated in the analysis.

Acknowledgments

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